A Note on Transmuted Inverse Power Lomax Distribution and Application to Breast Cancer Data

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Abstract
The Transmuted Inverse Power Lomax (TIPL) Distribution has been derived using Inverse Power Lomax Distribution (IPL) distribution and the Quadratic Rank Transmutation Map (QRTM). The developed distribution is more flexible and adaptable in modeling data exhibiting different shapes of the hazard function than its sub-models and other competing distributions. The mathematical expressions and shapes of the distribution function, probability density function, hazard rate function and reliability function are studied. The parameters of the TIPL distribution is estimated by the method of maximum likelihood. Finally, the TIPL distribution is applied to breast cancer data set and found to be better fit than IPL distribution and Inverse Lomax (IL) distribution.

Keywords: *TIPL distribution, QRTM, Reliability Function, Hazard Rate Function, Maximum Likelihood.*

1.0 Introduction

Transmuted distributions have been discussed widely in large scale experimental statistical data for model selection. In applied sciences such as medicine, engineering, seismography etc. modeling and analyzing experimental data are very important. Several distributions have been developed to model such kind of experimental data. The procedures used in such a statistical analysis depend heavily on the assumed probability model or distributions. That is why the development of large classes of standard probability distributions along with relevant statistical methodologies has been expanded. However, there still remain many important problems where the real data does not follow any of the classical or standard probability models.

Now a days transmuted distributions and their mathematical properties are widely studied for applied sciences experimental data sets. Transmuted Rayleigh Distribution by Merovci (2013), Transmuted Inverse Rayleigh Distribution was developed by Ahmad et al. (2014), Transmuted Generalized Inverse Weibull Distribution by Khan and King, 2013), Transmuted Modified Inverse Weibull Distribution was investigated Elbatal (2013), Transmuted Log-logistic Distribution by Aryal (2013), Transmuted Modified Weibull Distribution & Transmuted Lomax Distribution was studied by Ashour and Eltehiwy, (2013), Transmuted Fréchet Distribution by Mahmoud and Mandouh (2013), Transmuted Pareto Distribution by Merovci & Puka (2014), Transmuted Generalized Gamma Distribution developed by Lucena et al. (2015), Transmuted Weibull Lomax Distribution by Afify et al. (2015) are reported with their various structural properties including explicit expressions for the moments, quantiles, entropies, mean deviations and order statistics. All

the above transmuted distributions are derived by using Quadratic Rank Transmutation Map (QRTM) studied by Shaw and Buckley (2007). Report reveals that some properties of these distributions along with their parameters are estimated by using maximum likelihood and Bayesian methods. Usefulness of some of these new distributions are also illustrated with experimental data sets.
A random variable X has an IPL distribution if its cumulative distribution function (cdf) takes the form

$$G(x;\rho,b,c)=\left(1+\frac{x^{-\rho}}{b}\right)^{-c},x>0;\ \rho,b,c>0 \tag{1}$$

The corresponding probability density function (PDF) is

$$g(x;\rho,b,c)=\frac{\alpha c}{b}x^{-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-c-1},\ \ x>0;\ \ \rho,b,c>0 \tag{2}$$

The survival and hazard rate functions of the IPL distribution are, respectively,

$$S(x;\alpha,\rho,\lambda)=1-G(x;\alpha,\rho,\lambda)=1-\left(1+\frac{x^{-\alpha}}{b}\right)^{-c} \tag{3}$$

and

$$h(x;\alpha,\rho,\lambda)=\frac{g(x;\dot{\rho},b,c)}{S(x;\rho,b,c)}=\frac{\alpha\rho x^{-\alpha-1}\left(1+\frac{x^{-\alpha}}{b}\right)^{-c-1}}{\lambda\left\{1-\left(1+\frac{x^{-\alpha}}{b}\right)^{-\rho}\right\}} \tag{4}$$

Where c and ρ are top positive shape parameters and b is a scale parameter. to further enhance the area of applications of the IPL distribution, the model was extended by the addition of one parameter using the Quadratic transmutation map developed by Shaw and Buckley (2007).

2.0 Transmuted Inverse Power Lomax distribution

A random variable X is said to have a transmuted quadratic distribution if its cumulative distribution function (cdf) is given by

$$F(x)=(1+\lambda)G(x)+\lambda G(x)^2,\quad |\lambda|\le 1 \tag{5}$$

Where $G(x)$ is the cdf of the transmuted distribution and $F(x)$ is the cdf of the baseline distribution. Differentiating (1) w.r.t. X, we have the probability density function (pdf) of the transmuted distribution as

$$f(x)=g(x)[1+\lambda-2\lambda G(x)],\qquad |\lambda|\le 1 \tag{6}$$

Where $f(x)$ and $g(x)$ are represent the pdf of $F(x)$ and $G(x)$ respectively. It is observed that at $\lambda=0$, (2) will revert to the base distribution of the random variable X. inserting (1) and (2) in (5), we obtain the CDF of the Transmuted Inverse Power Lomax distribution given by

$$F(x)=(1+\lambda)\left(1+\frac{x^{-\rho}}{b}\right)^{-c}+\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-2c},\quad |\lambda|\le 1 \tag{7}$$

And the corresponding PDF as

$$f(x)=\frac{\alpha c}{b}x^{-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-c-1}\left[1+\lambda-2\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-c}\right],\qquad |\lambda|\le 1 \tag{8}$$

An expression for the survival and the hazard functions are respectively, given by

$$S(x)=1-\left[(1+\lambda)\left(1+\frac{x^{-\rho}}{b}\right)^{-c}+\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-2c}\right], \tag{9}$$

and

$$h(x) = \frac{\frac{\alpha c}{b} x^{-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-c-1}\left[1+\lambda-2\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-c}\right]}{1-\left[(1+\lambda)\left(1+\frac{x^{-\rho}}{b}\right)^{-c}+\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-2c}\right]} \tag{10}$$

the graphs of the CDF and the PDF are given in figures 1.0 and 2.0 for various values of the parameters of the TIPL distribution.

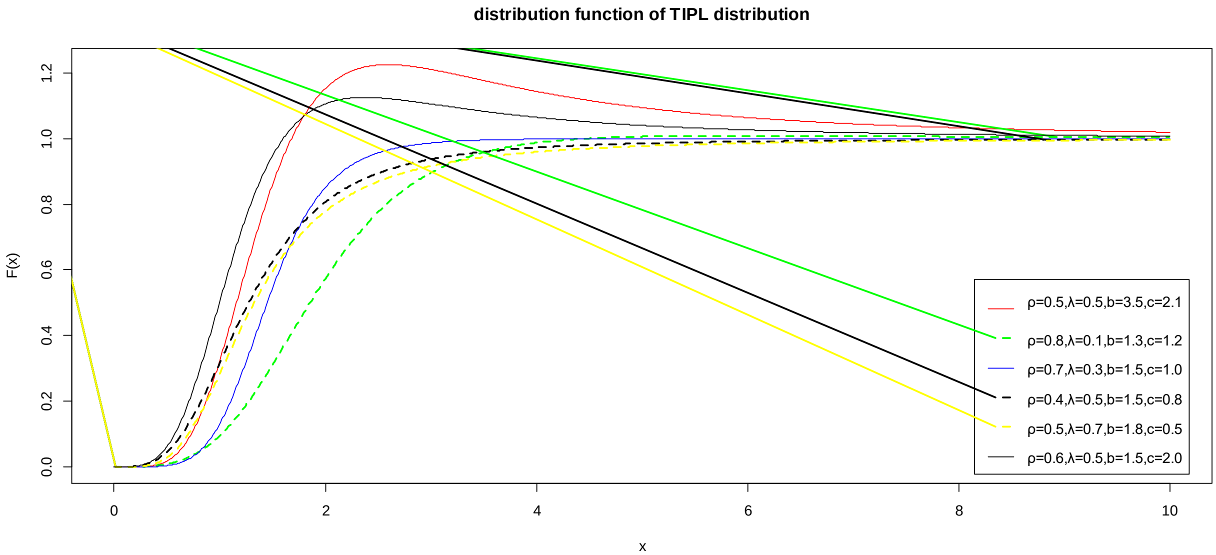

Figure 1.0. Graph of the distribution function of the $TIPL$

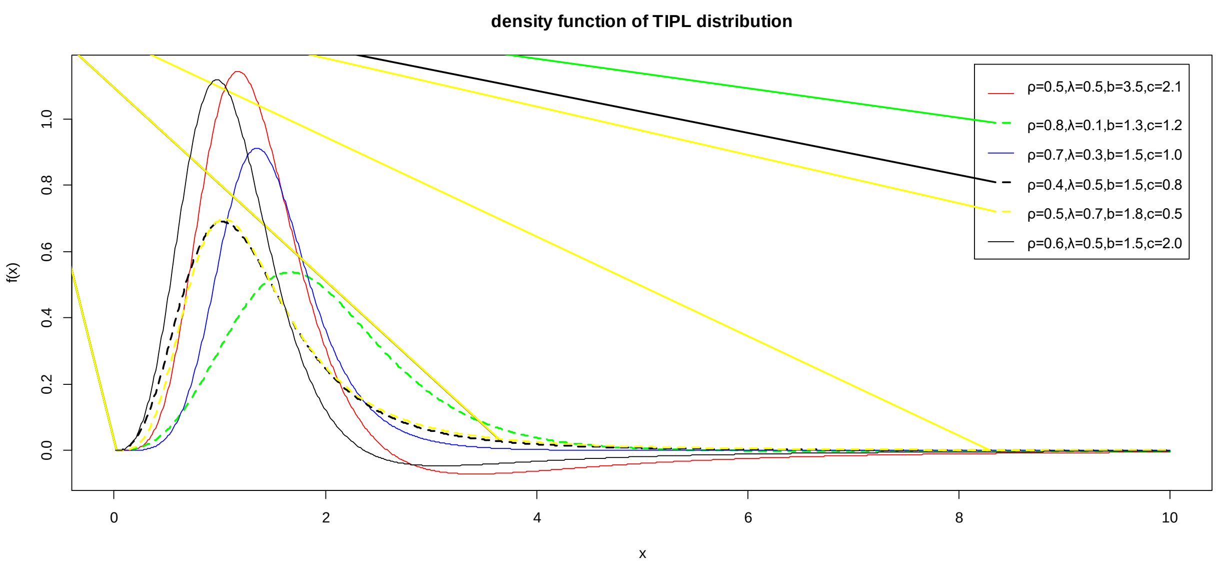

Figure 2.0. Graph of the distribution function of the $TIPL$

3.0 Moments of $TIPL$ distribution

The r^{th} moment of a $TIPL$ for random variable X can be obtained as

$$E(X^r) = \int_0^\infty x^r f(x)dx \tag{11}$$

Putting (6) in (9), we have

$$E(X^r) = \frac{\rho c}{b}\int_0^\infty x^{-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-c-1}\left[1+\lambda-2\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-c}\right]dx \tag{12}$$

By algebraic manipulation, we have

$$E(X^r) = \mathcal{H}_1 + \mathcal{H}_2 \tag{13}$$

where,

$$\mathcal{H}_1 = \frac{\rho c}{b}(1+\lambda)\int_0^\infty x^{r-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-c-1}dx \tag{14}$$

and

$$\mathcal{H}_2 = -2\frac{\rho c}{b}\lambda\int_0^\infty x^{r-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-2c-1}dx \tag{15}$$

By letting $y = \frac{x^{-\rho}}{b}$, in (14) and (15), finally, we have

$$\mathcal{H}_1 = c(1+\lambda)b^{-r/\rho}B\left[2-{}^{r}/_{\rho}, c-{}^{r}/_{\rho}\right] \tag{16}$$

And

$$\mathcal{H}_2 = -2c\lambda b^{-r/\rho}B\left[2-{}^{r}/_{\rho}, c-{}^{r}/_{\rho}\right] \tag{17}$$

Subsequently, we obtain an expression for the r^{th} moments of the $TIPL$ distribution as

$$E(X^r) = cb^{-r/\rho}\left\{(1+\lambda)B\left[2-{}^{r}/_{\rho}, c-{}^{r}/_{\rho}\right] - \lambda B\left[2-{}^{r}/_{\rho}, c-{}^{r}/_{\rho}\right]\right\} \tag{18}$$

By taking the value of $r = 1,2$. Then we obtain the first and the second central moments.

4.0 Maximum likelihood Estimates of the parameters

The maximum likelihood approach is used to estimates the unknown parameters of the distribution. Let $\underline{x} = x_1 \dots, x_n$ represent a random sample obtained from the TEIE distribution. The likelihood function $L(x;\ \zeta)$ and the log-likelihood function $logL(x;\zeta) = l(x;\zeta)$ corresponding are respectively given as

$$L(x;\zeta) = \frac{\alpha c}{b}\prod_{i=1}^{n} x^{-\rho-1}\left(1+\frac{x^{-\rho}}{b}\right)^{-c-1}\left[1+\lambda-2\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-c}\right] \tag{19}$$

and

$$l = \log\left(\frac{\alpha c}{b}\right) - (\rho+1)\sum_{i=1}^{n}\log(x) - (c+1)\sum_{i=1}^{n}\left(1+\frac{x^{-\rho}}{b}\right) + \sum_{i=1}^{n} log\left[1+\lambda-2\lambda\left(1+\frac{x^{-\rho}}{b}\right)^{-c}\right] \tag{20}$$

The log-likelihood expression given in (20) can be maximized using Adequacy Model in R.

4.1 APPLICATION

In this section, we compare the fit of the $TIPL$ model and some other competing models using breast cancer data sets. We measure how well the $TIPL$ distribution performs compared to Inverse Power Lomax and the Inverse Lomax (IL). The data represent 121 breast cancer patients' survival times during a specific period from 1929 to 1938. The data source is Ramos et al. (2013) and Tahir et al. (2014) studied these datasets. The observations are listed as follows: 0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4,10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7,16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1,30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0,38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0,43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0,51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0,111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0,154.0. For each model, we obtained the estimate of the parameters by using the maximum likelihood method and assessed the goodness-of-fit by using the following information criteria: Akaike information criterion (AIC), Consistent Akaike information criterion (CAIC), Cramer-von Mises (CM) statistic. In general, the smaller the value of the information criteria, the better the model fit to the data. Table 1 represent the exploratory data analysis of the breast cancer data which shows that the data is positively skewed, over-dispersed, and leptokurtic. Figure 3.0 is the Box, the Total Test on Time (TTT) and violin plots for the neck cancer data which also shows that the data is positively skewed. is given in Figure 4.0 which indicates that the cancer data exhibits non-monotone (bathtub) failure rate.

Table 1.0 Exploratory data analysis of the breast cancer data

Data	n	Range	Lower quartile	Median	Upper quartile	mean	Var.	Skew.	Kurt.
$Data$	121	153.70	17.50	40.0	60.0	46.33	1244.46	1.04	3.40

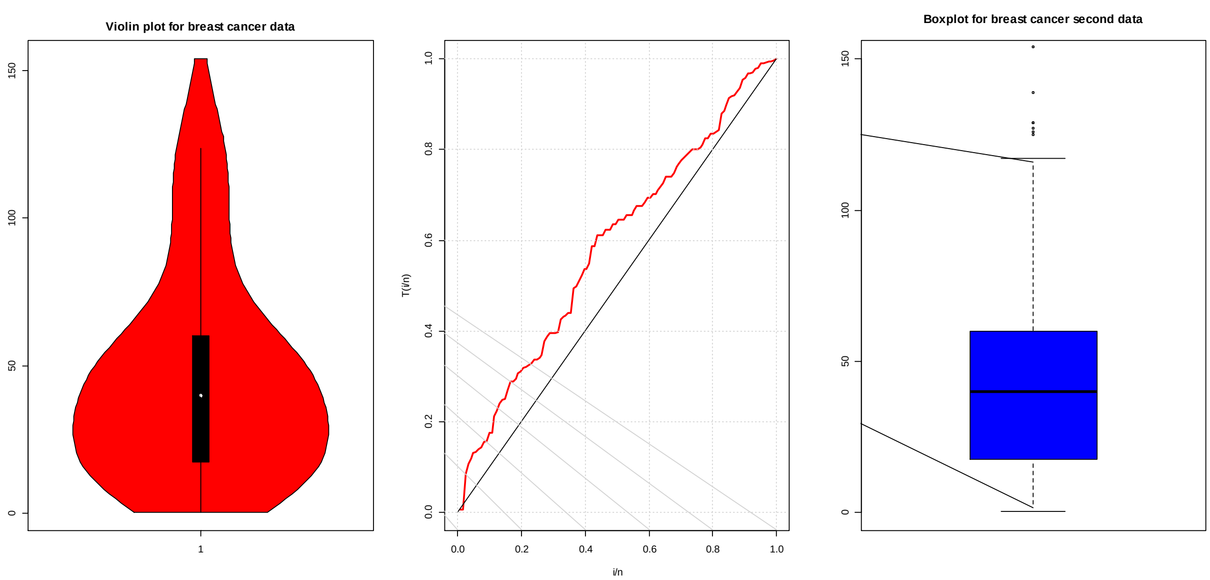

Figure 3.0 graph of the Violin plot, TTT plot and Box plot for breast cancer data

Table 2. MLEs and standard error (in braces) and measures of goodness of fit for the breast cancer data.

	$Parameters\ Estimation$				$Measures\ of\ goodness\ of\ fit$				
$Model$	ρ	b	c	λ	l	AIC	$CAIC$	$HQIC$	BIC
$TIPLD$	1.7415 (0.0707)	0.0030 (0.0007)	0.5475 (0.3332)	0.8437 (0.204)	587.11	1182.21	1182.56	1186.76	1193.39
IPL	1.6952 (0.060)	0.0025 (0.0004)	1.0 (0.1254)	$-$ $(-)$	588.31	1182.62	1182.82	1186.02	1191.0
IL	0.0775 (0.0225)	$-$ $(-)$	2.2508 (0.4954)	$-$ $(-)$	605.93	1215.86	1215.96	1218.13	1221.45

It will be observed from values of AIC, CAIC, HQIC, and BIC for $TIPLD$, IPL and the IL from Table 2 that $TIPL$ gives lower values. Hence the $TILP$ provides the best fit among the the model considered.

5.0 Conclusion

The Transmuted Exponentiated Inverse Exponential distribution has been generated and their parameters are estimated. The developed distribution is applied to breast cancer data and it is compared with Transmuted Inverse Exponential and the Inverse Exponential distribution and leads to better fit than the other two distributions. Hence the new Transmuted Exponentiated Inverse Exponential distribution can be applied in the field of medicine.

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